ESCUELA DE CONSTRUCCIÓN CIVIL
FACULTAD DE INGENIERÍA

Revista de la Construcción
Journal of Construction

Research Article

A criticism on strengthening glued laminated timber beams with fibre reinforcement polymers, numerical comparisons between different modelling techniques and strengthening configurations

Dilan Çankal[1,*], Gökhan Şakar[2], Hüseyin Kürşat Çelik[3]

[1] Department of Civil Engineering, Manisa Celal Bayar University, Manisa (Turkey), dilan.cankal@cbu.edu.tr
[2] Department of Civil Engineering, Dokuz Eylül University, Izmir (Turkey), gokhan.sakar@deu.edu.tr
[3] Graduate School of Natural and Applied Sciences, Dokuz Eylül University, Izmir (Turkey), celik.huseyinkursat@ogr.deu.edu.tr
*Correspondence: dilan.cankal@cbu.edu.tr

Received: 31.07.2023; **Accepted:** 23.11.2023; **Published:** 29.12.2023

Citation: Çankal, D., Şakar, G., and Çelik, H. K. (2023). A criticism on strengthening glued laminated timber beams with fibre reinforcement polymers, numerical comparisons between different modelling techniques and strengthening configurations. Revista de la construcción. Journal of construction, 22(3), 661-678. https://doi.org/10.7764/RDLC.22.3.661.

Abstract: The application of CFRP for strengthening timber structures has proven its efficiency in enhancing load-bearing capacity and, in some cases, the stiffness of structural elements, thus providing cost-effective and competitive alternatives both in new design and retrofitting existing historical buildings. In this study, glued laminated timber beams strengthened with CFRP were examined. A number of test beams with different reinforcement configurations and beam sizes were selected from the literature. These beams were analysed with three different methods as numerical, theoretical and code perspective. For the numerical method, a 3D nonlinear finite element model which includes damage and fracture mechanics was constructed. All test beams were studied with different methods and results were compared with respect to initial flexural stiffness, midspan vertical displacement and failure load. The effectiveness of methods and strengthening configurations were stated and suggestions for practical application of FRP-strengthened timber beams were presented.

Keywords: Glued laminated timber beam, FEM, CFRP, strengthening, modelling.

1. Introduction

Timber and other wood products have always been favourable constructional materials throughout the history. This case originates from several properties of timber comparing alternative structural materials like steel and concrete. Obtaining structural elements from wood and tree is relatively easy. Energy consumption required for the finished elements is low and it has high strength to weight ratio. With all its basic properties, timber is used as a building element in various ways with respect to its place and purpose.

Although timber has many advantages; it is inherently a material with defects which cause some unexpected disadvantages and limit the use of its potential (Gustafsson, 2003; McConnell et al., 2014). Very well-known examples of these defects are knots, sensitivity to moisture and being an anisotropic material. Defects lead to decrease in the strength of the timber. That is why industrial wood products come forward as a better solution. These materials are produced with technology and industrial opportunities; hence, the disadvantages of timber could be eliminated. Currently, many wood products are used in construction works and plywood, cross laminated timber and glulam are well known examples. Glulam, also known as glued laminated timber, has a special place in wood products. It is an old and pioneer wood product and has a wide range of applications. Glued laminated timber is laminating solid timber elements with glue. Solid timber parts are taken so that they minimise the existing defects and create an element with better properties. Thus, laminated products can be used in applications such as theatres, bridges and multi-storey buildings that require much larger dimensions where the use of solid wood is not possible. At the same time, making the best use of low-class wooden elements that are not suitable for use contributes to more efficient use of raw material resources (Aydın et al., 2004).

Strengthening the elements with a more rigid material is a good solution to keep displacements under control when crossing large span and to limit the dimensions of the structural member for aesthetic purposes (Haiman & Žagar, 2002). Until the 20th century, bars and plates made of metallic elements such as aluminium and steel were frequently used as reinforcement of structural elements (Bulleit et al., 1989; Meier, 1995). Towards the end of the 20th century, fibre reinforced polymer materials (FRP) with a lower density than steel began to be used. Despite its low density, FRP has very good mechanical properties and corrosion resistance is higher than steel (Ali, 2018; Lee et al., 2019). For this reason, in recent years, the use of FRP has been more focused on the use of FRP in the reinforcement of laminated timber beams (Donadon et al., 2020; Lacroix & Doudak, 2018; Micelli et al., 2005).

The role of FRP, which has good mechanical properties, is to limit local cracks and reduce crack opening, as well as improve the mechanical properties of timber locally. Laminated timber beams tested in flexural often take damage at joints, near defects, joints or in the region of maximum stress (Issa & Kmeid, 2005; Li et al., 2009). FRP is applied in these regions to obtain a more predictable plastic failure mode (Thorhallsson et al., 2017). Corradi et al., (2021) conducted a series of experiments to examine the structural response of timber members locally reinforced with FRP plates after re-exposing to flexural loads where defects were recorded in timber beams after static flexural. It was shown that it is possible to partially restore the flexural capacity of damaged timber beams by applying the reinforcement method proposed in the study.

With the FRP reinforcement of the laminated timber beams tested in flexural, a more ductile failure occurs compared to the brittle behaviour, resulting in an increase in the stiffness of the beams (Alhayek & Svecova, 2012). Many parameters can be mentioned that affect this improvement in the behaviour and mechanical properties of laminated timber beams. The performance of beams also changes with the difference of many parameters such as the variety of glue between laminate layers, the interaction of the interface of laminated timber and FRP material, and the choice of different types of FRP and configurations (Gáborík et al., 2016; Morales-Conde et al., 2015). The flexural behaviour of laminated timber beams reinforced with FRP has been studied by various researchers (Gilfillan et al. 2001; Raftery & Harte, 2011). Parameters affecting the behaviour of FRP reinforced laminated timber are summarised in Figure 1 in general terms.

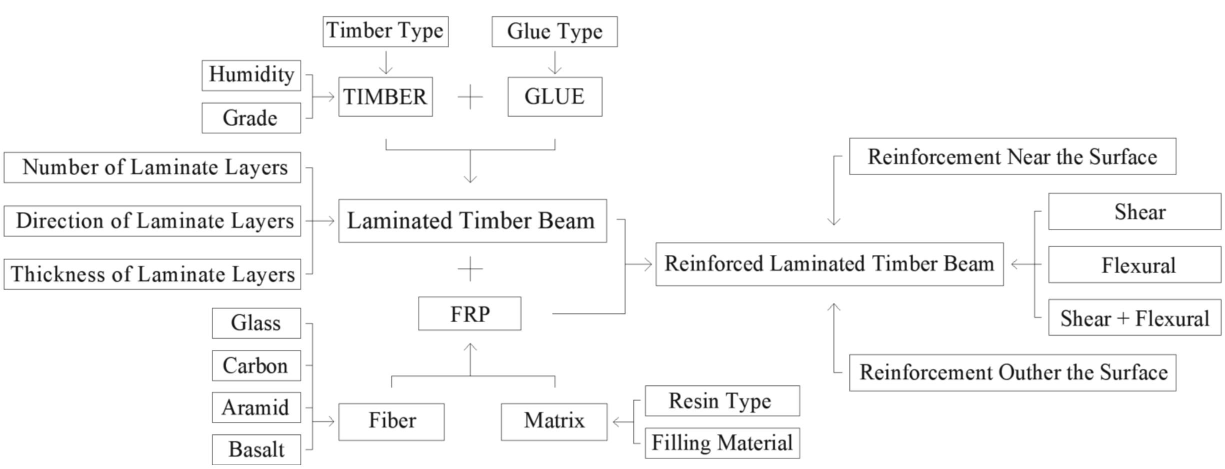

Figure 1. Scheme of possible reinforcement of laminated timber beam.

In a parametric study conducted by Kim & Harries, (2010), the variables were selected as timber type, elasticity modulus of CFRP and reinforcement ratios. The change in the modulus of elasticity of the CFRP material did not significantly affect the failure loads of the timber beams. In another study investigating the reinforcement of laminated timber beams produced from low, medium and high quality timbers with FRP in the tensile zones, the greatest increase in strength was obtained in lower timber grades. It was noted that the highest value-added benefits after FRP reinforcement can be realised in lower timber grades, which have a larger difference in relative tensile/compressive strength values (Dagher et al., 1996).

The positions, widths, thicknesses, lengths, and numbers of FRP reinforcements can vary according to the beam. Johns & Lacroix, (2000) investigated the layer and length effect of FRP reinforced in a U shape by wrapping a certain part of the beam height in the tensile zone of the timber. Double layer FRP reinforcement was used for small-scale and single-layer FRP reinforcement for large scale beams. Compared to control beams, strength increases of between 40% and 70% were obtained depending on the FRP length in FRP reinforced beams. Most of the damage to the beams occurred as a result of plastic deformation of the timber material rather than the composite material. It was confirmed that FRP reinforced into timber is highly effective on the behaviour of low-quality timber. It was observed that the expansion of the cracks formed was prevented, the stress densities around the defects decreased and the propagation stopped. Borri et al., (2005) compared beams reinforced with two different FRP fabrics centred on the tensile zone and surrounding part of the lower corners of the beam. In the comparison of beams with a reinforcement ratio of 0.082%, the maximum load increase and stiffness in the CFRP reinforcement in the middle of the beam were 42.3% and 22.5%, respectively, while these increases were 55% and 30.3% with another method. Bakalarz & Kossakowski, (2022) studied the flexural behaviour of beams reinforced with different types of FRP along the soffit alone and on both the soffit and side surfaces. The highest increase in ductility was obtained with FRP reinforcement on both the bottom and side surfaces of the beams.

2. Literature review

2.1. *Theoretical method studies*

It is possible to predict the flexural behaviour of laminated timber beams reinforced with FRP by various theoretical methods. Theoretical methods are generally based on a set of equations based on the Euler-Bernoulli beam theory (Wang et al., 2000). These methods include linear or non-linear analysis of beams with elastic, elastic-plastic or plastic material behaviour assumptions. Since timber is an anisotropic material, its failure modes are uncertain, but different behaviour is expected in tension and compression zones. Fiorelli & Dias, (2003) analysed timber beams reinforced with various FRPs both experimentally and theoretically. FRP was reinforced with adhesive to the tension areas of timber beams. Different theoretical models have been proposed according to different failure modes. The ratios of flexural stiffness values of experimental and theoretical models vary between 0.99 and 1.08. It has been observed that the flexural stiffness values obtained from the theoretical models are mostly lower than the values obtained experimentally. This provides benefits in terms of structural safety. The model was based on elastic-plastic behaviour in compression while elastic brittle behaviour in the tension of wood. Timbolmas et al., (2022) developed a comprehensive analytical model that takes into account different elastic modulus values and different elastic behaviours in tensile and compression zones of CFRP-reinforced laminated timber beams. These models are based on the equilibrium, constitutive, and compatibility equations for non-reinforced and FRP-reinforced beams separately. The developed method is useful for retrofitting existing structures by allowing an estimation of the degree of improvement required for a structural beam member.

Borri et al., (2005) analysed the CFRP reinforcement to the tension zones of timber beams in different configurations with a nonlinear theoretical model and experimental method. With the proposed theoretical model, satisfactory results were obtained in the estimation of the failure loads of the beams. In addition, the FRP reinforcements proposed in the study were very useful for on-site intervention applications encountered in strengthening historical timber structures. This model also considered the elastic-plastic behaviour of wood. Micelli et al., (2005) proposed a theoretical method for the reinforcement of CFRP bars to predict increases in the ultimate load carrying capacity and stiffness of unreinforced laminated timber beams. In the theoretical model, the material properties were defined with a linear stress distribution along the section depth and an ideal stress-strain, assuming that the connection between timber and FRP bars is compatible. Experimental results were compared with numerical results that showed good agreement in terms of load and displacement values. Morales-Conde et al., (2015) proposed analytical methods for two different configurations, namely repairing and strengthening of damaged beam ends and centre. In this method, visual advantages were provided by the FRP reinforcement into the timber beams, while an increase of more than 50% was observed in the approximate load carrying capacity.

2.2. *Finite element method (FEM) studies*

Analytical methods can be efficient for simple elements. However, it is hard to study local behaviour and complex structural geometry with analytical methods. One of the most widely used numerical methods for solving complex problems is FEM, which many researchers prefer to verify experimental studies. Software such as ANSYS (ANSYS, 2011) and ABAQUS (Systemes, 2012), which are based on FEM and can solve problems quickly and are highly preferred. There are many important parameters in the analysis of FRP reinforced timber and laminated timber beams subjected to flexural by the FEM, such as the bond between the timbers forming the laminated timber beam, the connection between the FRP and timber, the support boundary conditions, the fracture criteria and the models of the material behaviour. If these parameters are defined as close to reality and accurate numerical models are developed, it is possible to optimise such systems (Raftery & Harte, 2013). It is a fast and economical method especially in examining the effect of reinforcement percentage in strengthening with FRP.

Raftery & Harte, (2011) investigated the effect of the FRP plate reinforced at the bottom of the beam and between the last two laminated layers using the ANSYS program. In the numerical model, anisotropic plasticity theory and maximum stress criterion failure model were used in the compressive zone. A good correlation was obtained between predicted behaviour and experimental study results. It has been observed that FRP plate reinforcement between laminated timbers significantly improved the performance of the beam. Yahyaei-Moayyed & Taheri, (2011) investigated the short term and creep performances of timber beams reinforced with aramid fibre reinforced polymer (AFRP) sheet. A nonlinear FEM model was employed to calculate the creep response of AFRP reinforced timber beams. It was stated that the analysis results showed good agreement with the experimental results performed on beams reinforced with AFRP.

Kim & Harries, (2010) developed a three-dimensional finite element analysis model based on the orthotropic properties of timber. Different failure modes, load-displacement relationships, strain developments and stress concentrations of CFRP reinforced timber beams consisting of different types of timber were investigated. A notch was created in the critical zones of some beams to simulate possible damage. Thus, the reducing effect of the FRP reinforcement on timber beams has been considered. The externally bonded CFRP improved the energy absorption capacity of the damaged timber beams. However, damaged beams have been observed to have lower reserve strength than undamaged beams, even when reinforced with CFRP composites. İşleyen et al., (2021) investigated reinforced with anchored and unanchored CFRP strips on the tensile zones of laminated timber beams with ABAQUS, considering different parameters. The Hashin damage model, in which the nonlinear behaviour of timber is taken into account, was used to evaluate the different failure modes. Good accordance was obtained between the results obtained by the FEM and the experimental results.

2.3. *Code and standard studies*

Building design codes and standards play a major role in practical engineering. They are a useful source and guide for designers with respect to literature knowledge. Nowadays, several codes and standards are available for both structural timber and FRP strengthening works. *Eurocode 5: Design of Timber Structures* (European Committee, 2004), *National Design Specification of Wood Construction* (American Wood Council, 2015) and *American Concrete Institute (ACI)-Committee 440: Guide for the Design and construction of Externally Bonded FRP Systems for Strengthening Concrete Structures* (ACI Committee 440, 2017) are well-known examples. However, there is very limited source available in codes and standards for strengthening timber elements with FRPs. Moreover, the preliminary study of the *National Research Council of Italy* on the subject is nearly the only chance to consider both timber and FRP in the same document (National Research Council, 2005).

From the perspective of practice, several points should be illuminated for applications; how to analyse and design. In general, structural analysis is performed for determining element design forces and analytical or other methods are used for element load bearing capacity calculations. At this point, there is very limited guidance in codes and standards for analysing timber elements, considering literature knowledge. In general, linear-elastic first-order analysis is suggested. Eurocode 5 allows designers to employ elastic-perfectly plastic behaviour only if elements will experience solely compression. Frame or plane elements are suggested depending on the problem and the element types. A similar expression is valid for strengthening the FRP case. Codes and standards are more focused on design considerations.

Critics of timber design codes focus on the scope. Several studies emphasised that EN5 has a limited scope to evaluate today's building variation (Dietsch & Winter, 2018). Both the connection and timber elements are limited compared to available market products. Furthermore, the lack of documentation about strengthening timber elements was also highlighted in some papers (Dietsch, 2016). Cost comparing studies were another area of research on timber design codes (Wacker & Groenier, 2010). Experimental and theoretical studies are favourable for more realistic and useful codes and continuously reported (Aloisio et al., 2023; Theiler et al., 2013). Similar studies have been done for FRP strengthening as well. Although many studies have been performed on both timber and FRP strengthening, there is a lack of knowledge on FRP strengthening timber elements from the perspective of codes and standards.

This study was conducted to close the gap between literature knowledge and engineering practice. For this, a set of sample beams and strengthening strategies was created. A non-linear 3D FEM was developed to represent experimental results and analytical analysis was performed in place of member capacity calculations. Then, a 3D linear-elastic finite element analysis was performed with anisotropic frame and plane stress elements to represent code-based design practice. Samples were analysed separately with these three methods, resulting in the failure load, midspan vertical displacement and initial flexural stiffness.

3. Material and method

3.1. *Material properties*

The physical and mechanical properties of FRP and timber constituting the laminated timber beam are taken from the study of Glišović et al., (2016). Timber was graded as C24 according to EN 338 (BSI, 2009) and its properties were given in Table 1. The properties of the carbon fibre polymer (CFRP) plates selected as the reinforcement element were given in Table 2. It was assumed that melamine-urea-formaldehyde (MUF) type adhesive was used for the bonding between the layers of timber (Tran et al., 2014). Considering its good creep, high mechanical properties and toughness, a type of adhesive such as epoxy was assumed between the timber and CFRP (Raftery et al., 2009).

Table 1. Properties of timber (Glišović et al., 2016).

Property	Value	CV
Tensile strength parallel to grains (MPa)	27.8	25.2%
Compressive strength parallel to grains (MPa)	36.3	9.8%
Modulus of elasticity parallel to grains (MPa)	11080	12.6%
Flexural strength (MPa)	42.5	20.6%
Type	Spruce	
Density (kg/m^3)	427	

Table 2. Properties of CFRP plate (Glišović et al., 2016).

Property	Value	CV
Tensile strength (MPa)	2846	4.5%
Modulus of elasticity (MPa)	165543	2.8%
Strain at break (%)	1.73	3.2%
Density (g/cm^3)	1.6	

Timber and CFRP are fibrous and anisotropic materials. The properties of anisotropic materials vary according to the direction and distribution of the fibres along the applied load. For this reason, it is useful to carry out many tests when determining the properties of anisotropic materials. The coefficients of variation (CV) in the Tables 1 and 2 were determined using the values in these tests.

3.2. *Methodology of analysis*

The dimensions of the laminated timber beams were chosen considering the studies in the literature and the possible dimensions to be used in field applications. A total of six laminated timber beams to be analysed were divided into two groups. Group 1 laminated timber beam dimensions are 70x90x1500 mm, while Group 2 dimensions are 50x90x1500 mm. All beams have three laminated layers, with each laminated layer being 30 mm high. The effect of the CFRP area surrounding the width of the beams was investigated by changing only the widths of the cross-sections of the beams. The geometric values of all beams analysed by the numerical analysis were given in Table 3. The notation of the beams was determined according to the beam and CFRP cross-section widths. T70 and T50 are non-reinforced reference beams of Groups 1 and 2, respectively. For reinforced beams, C notation was given with CFRP width values.

Table 3. All analysed beams.

Notation of beams	Width of beam (mm)	Height of beam (mm)	Width of CFRP (mm)	Thickness of CFRP (mm)
T70	70	90	0	0
C35-T70	70	90	35	0.5
C70-T70	70	90	70	0.5
T50	50	90	0	0
C20-T50	50	90	20	1
C35-T50	50	90	35	1

CFRP plates were reinforced on the outer surface in the tension zone to the beams obtained with the specified properties and dimensions. All beams were analysed according to theoretical analysis, finite element analysis and code-based analysis, based on the four-point flexural test setup shown in Figure 2. As a result of each method, the moment carrying capacity, failure load value and initial flexural stiffness of the beams were determined. The results obtained together with the principles of the methods were evaluated.

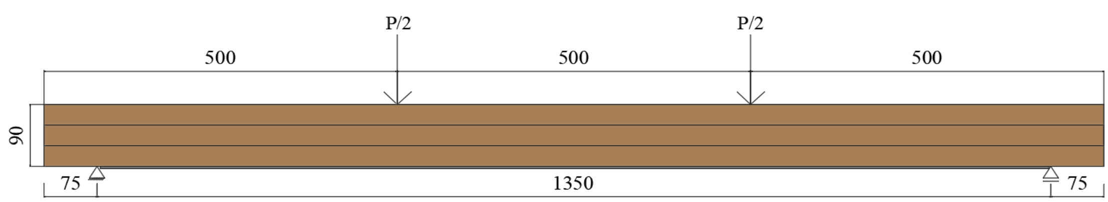

Figure 2. Flexural test setup of reinforced beam.

3.3. *Theoretical method*

In the theoretical model described here, it was assumed that the plane sections remain plane under the flexure effect and the sections perpendicular to the beam axis then remain orthogonal (Hoseinpour et al., 2018). In addition, it was assumed that timber exhibits linear elastic behaviour in the tension zone, linear elastic-perfectly plastic behaviour in compression, and the modulus of elasticity is the same for linear elastic behaviour in both zones. Strain and stress distributions in cross-section for these behaviours from beams were shown in Figure 3.

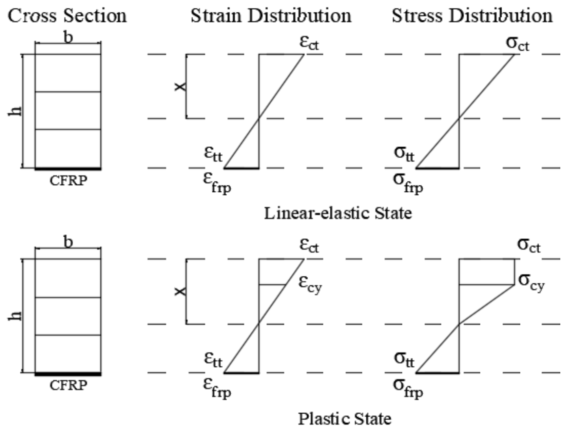

Figure 3. Idealised strain and stress distributions of CFRP reinforced laminated timber beams.

In the compression zone, if the strain of timber at the yield stress (ε_{cy}) reaches the strain of timber at the compressive stress (ε_{ct}), the linear elastic state exists. In this case, before the compressive stress of timber (σ_{ct}) reaches the maximum strength, if the tensile stress (σ_{tt}) in the outermost timber has reached the ultimate strength, the damage has occurred in the section, and the maximum strength value corresponds to the flexural strength (σ_t). Where the width of the laminated timber beam (b), the length of the laminated timber beam (h), the cross-sectional area of CFRP (A_f), the ratio of the modulus of elasticity of CFRP to the modulus of elasticity of timber (n), and the beam span (l), the moment of inertia (I), neutral axis distance (x_e), ultimate moment (M_{ue}), and failure load (P_e) in the linear elastic state were calculated using Eqs. (1), (2), and (3), respectively.

$$I = \frac{bh^3}{12} + bh\left(\frac{h}{2} - x_e\right)^2 + nA_f(h - x_e)^2 \tag{1}$$

$$x_e = \frac{1}{bh + nA_f}\left(\frac{bh^2}{2} + nA_f h\right) \tag{2}$$

$$M_{ue} = \sigma_t\left(\frac{I}{h - x_e}\right) = \frac{P_e}{2}\frac{l}{3} \tag{3}$$

In the compression zone, if the strain of timber at the compressive (ε_{ct}) reaches the strain of timber at the yield strength (ε_{cy}), the plastic state exists. In this case, before the tensile stress of timber (σ_{tt}) reaches ultimate strength if the compressive stress of the timber (σ_{ct}) has reached the maximum strength, damage has occurred in the section. Neutral axis distance (x_p), ultimate moment (M_{up}) and failure load (P_p) in the plastic state were calculated using Eqs. (4) and (5), respectively.

$$\sigma_{ct}\left[x_p - \frac{\sigma_{ct}}{\sigma_t}(h - x_p)\right] + \frac{{\sigma_{ct}}^2}{\sigma_t}\left(\frac{h - x_p}{2}\right) - \sigma_t\frac{(h - x_p)}{2} - \sigma_t\left(\frac{nA_f}{b}\right) = 0 \tag{4}$$

$$M_{up} = \frac{b}{6}\left[3\sigma_{ct}{x_p}^2 + \left(2\sigma_t - \frac{{\sigma_{ct}}^3}{{\sigma_t}^2}\right)(h - x_p)^2 + 6\sigma_t\frac{nA_f}{b}(h - x_p)\right] = \frac{P_p}{2}\frac{l}{3} \tag{5}$$

In this model proposed by Glišović et al., (2016), it was assumed that the adhesive used between timber laminated layers and between CFRP and timber has good mechanical properties. Accordingly, it is accepted that there are no slip and rupture in the bonding interfaces. The theoretical methods developed for the moment capacity of CFRP-reinforced laminated timber beams are based on similar assumptions (Fiorelli & Dias, 2011; Lu et al., 2015). Only the stress-strain relationship for CFRP is assumed to be linear elastic. However, with CFRP reinforcement, the tensile stress will increase significantly under the flexural effect. For this reason, a modification coefficient should be used in the final moment calculations of CFRP reinforced beams. Different researchers have optimised their proposed analytical methods using different modification factors that vary according to the reinforcement configuration they have chosen. Glišović et al., (2017) suggested the modification factor as 1.15 and 1.25, respectively, depending on the vertical and horizontal reinforcement of the CFRP plate. Gentile et al., (2002) suggested as 1.3 for the analysis of timber beams reinforced with GFRP bars. Yang et al., (2016) considered the modification factor as 1.4 in the proposed model for laminated timber beams reinforced with FRP and steel. Considering the reinforcement configuration within the scope of this study, the modification coefficient was taken into account as 1.25.

3.4. *Finite element method (FEM)*

Laminated timber beams reinforced with CFRP were analysed using the widely used ABAQUS program, which analyses using the finite element method. The timber layers beam and the CFRP plates were modelled as a 3D solid model with an 8-node linear geometric hexahedral solid element (C3D8). The elastic properties of timber and CFRP plate were given in Table 4 based on the local coordinate system given in Figure 4.

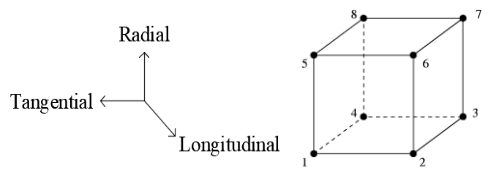

Figure 4. Local coordinate system and material model of timber and CFRP plate in FEM.

To define the elastic properties of timber in FEM, the longitudinal modulus of elasticity (E_L) was obtained from the experiment conducted by Glišović et al., (2017) in the direction parallel to the fibres. The modulus of elasticity in the radial direction (E_R) and the modulus of elasticity in the tangential directions (E_T) were calculated by Eq. (6) proposed by Bodig & Jayne, (1982). Shear modulus in the share planes (G_{LR}, G_{LT}, G_{RT}) were calculated by Eqs. (7) and (8), respectively. The poisson ratios $(\nu_{LR}, \nu_{LT}, \nu_{RT})$ were determined in accordance with the values suggested for softwood by Bodig & Jayne, (1982). The elastic properties of the CFRP plate were used in the study by Glišović et al., (2017). Elastic property values of timber and CFRP plate defined in FEM were given in Table 4.

$$E_L: E_R: E_T \approx 20: 1.6: 1 \tag{6}$$

$$G_{LR}: G_{LT}: G_{RT} \approx 10: 9.4: 1 \tag{7}$$

$$E_L: G_{LR} \approx 14: 1 \tag{8}$$

Table 4. Elastic properties of timber and CFRP in used FEM.

Elastic Property	Symbol	Timber	CFRP
Elasticity Modulus (MPa)	E_L	11080	165543
	E_R	886	10000
	E_T	554	10000
Poisson Ratio (-)	ν_{LR}	0.37	0.30
	ν_{LT}	0.42	0.30
	ν_{RT}	0.47	0.03
Shear Modulus (MPa)	G_{LR}	791	5000
	G_{LT}	744	5000
	G_{RT}	79	1000

CFRP plate was idealised as linear elastic with brittle failure mode. Assuming that the damage starts from the timber and progresses, the material properties of CFRP and timber were taken the same as the theoretical method. The expected plastic behaviour of laminated timber in the compression zone was modelled using the anisotropic theory of plasticity based on the transition condition to the plastic state proposed by Hill's criterion for orthotropic materials (Hill, 1948; Raftery & Harte, 2013). It represents a generalised version of von Mises's yield criterion that takes into account the anisotropy of the material's strength. Normal compressive yield stresses for the three orthogonal directions and yield shear stresses in the three shear planes were assumed to satisfy the criterion (Abrate, 2008; İşleyen et al., 2021).

The interfaces between CFRP and timber were modelled with the tie constraint, while the interfaces between the timber layers were modelled with the *Cohesive Zone Model (CZM)*. The nonlinear behaviour, damage and crack propagation that can occur in the bond between timber layers can be accurately simulated with CZM. The behaviour of this bond expresses progressive failure with a bilinear traction-separation law (Danielsson & Gustafsson, 2014; Lee et al., 2010). The linear elastic behaviour of the initial response before damage initiation is as in Eq. (9). The damage initiation criterion of the maximum stress is as in Eq. (10).

$$\{\sigma\} = \begin{Bmatrix} \sigma_n \\ \sigma_t \end{Bmatrix} = \begin{bmatrix} K_n & 0 \\ 0 & K_t \end{bmatrix} \begin{Bmatrix} \delta_n \\ \delta_t \end{Bmatrix} \tag{9}$$

$$\left(\frac{\sigma_n}{\sigma_n^c}\right)^2 + \left(\frac{\sigma_t}{\sigma_t^c}\right)^2 = 1 \tag{10}$$

When this criterion is reached, a damage variable is applied to the maximum stresses so that adhesive damage can be simulated as the progressive reduction of adhesive stiffness. While Mode I is determined according to the normal stress (σ_n) and its corresponding separation (δ_n) obtained by the modified Double Cantilever Beam (DCB) test, Mode II is determined according to the shear stress (σ_t) and its corresponding separation (δ_t) obtained by the pure shear test. The critical strengths (σ_n^c, σ_t^c), the initial stiffness (K_n, K_t) and the maximum separations ($\delta_n^{max}, \delta_t^{max}$) are the optimum cohesive parameters for mode I and II, respectively (Fortino et al., 2012; Khelifa et al., 2015). In this study, a mixed mode model, in which normal and shear stresses arise concurrently, was applied in the fracture zone. The optimum cohesive parameters suggested by (Tran et al., 2014) used for Mode I and Mode II are given in Table 5.

Table 5. Optimum cohesive parameters used for mode I and mode II (Tran et al., 2014).

	K_n (N/mm²/mm)	σ_n^c (N/mm²)	δ_n^{max} (N/mm²)
Mode I	4.5	1.6	0.005
	K_t (N/mm²/mm)	σ_t^c (N/mm²)	δ_t^{max} (N/mm²)
Mode II	30	9.7	0.00005

The load and support points of the beams are determined as shown in Figure 5. The boundary conditions of the supports are defined in accordance with the conditions of one fixed and the other movable support. The failure load (P_{FEM}) and corresponding midspan vertical displacement (w_{FEM}) values of the beams in FEM are found until failure occurs.

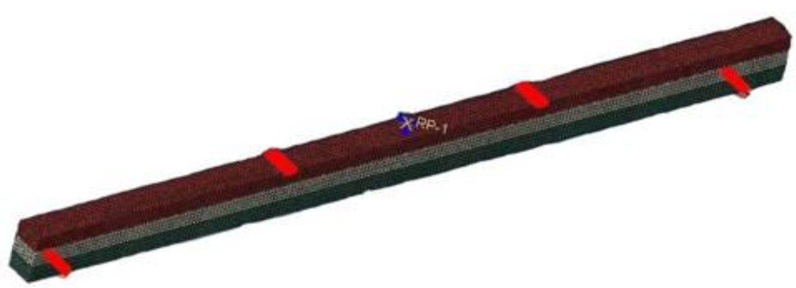

Figure 5. Beam with defined support and load points.

The load (P) and midspan vertical displacement (w) graphs were drawn with the obtained values. The energy absorption capacities of all beams until they reach the failure load values were found by calculating the area under the graphs. In addition, the initial flexural stiffness of the beams was determined by considering the linear elastic states of the load-midspan vertical displacement curves. Where the distance from the support to the loading point (α) and distance between supports (l), the initial flexural stiffness of beams (EI_{FEM}) in FEM were calculated using Eq. (11).

$$EI_{FEM} = \frac{\alpha \Delta P}{48 \Delta w}(3l^2 - 4\alpha^2) \tag{11}$$

The slope of the curve between the load in a certain range and the corresponding displacement values of the load-midspan vertical displacement graph in the elastic region is expressed as $(\frac{\Delta P}{\Delta w})$.(Glišović et al., 2016) used the slope of the load-displacement curve between 10% and 40% of the failure load when calculating the initial flexural stiffness value of the system. (Donadon et al., 2020) calculated the slope of the curve between 10% and 50% in his study. In this study, the slope of the curve between 10% and 40% of the failure load was considered.

3.5. *Codes and standards*

To represent engineering practice, an analysis model was developed. In this model, both timber and CFRP components were assumed as linear elastic up to failure. For analysis, material and section properties must be determined. Material and section properties were directly adopted from Tables 1, 2 and 3. With these parameters, the stiffness matrix of a frame or plane stress element can be obtained. Failure load was controlled according to stress, and the flexural stiffness and displacements were directly obtained from elastic theory.

Analysis was performed in two phases; first, frame elements were employed and then, layered-anisotropic elastic plane stress elements were preferred. In frame analysis, sections were modelled as composite, ignoring the interaction and slip between timber and CFRP. In plane elements; timber and CFRP layers were taken into account with assuming perfect bond between layers. CFRP layers were not modelled directly. To simplify, CFRP layers were transformed into equivalent timber layers according to their contribution to the inertial moment. The transformation process between CFRP-timber composite to pure timber section was done with respect to flexural stiffness since the problem is a beam flexural problem. Transformation and analysis schemes were given in Figure 6. All analyses were done with SAP2000 (Computers and Structures, 2015).

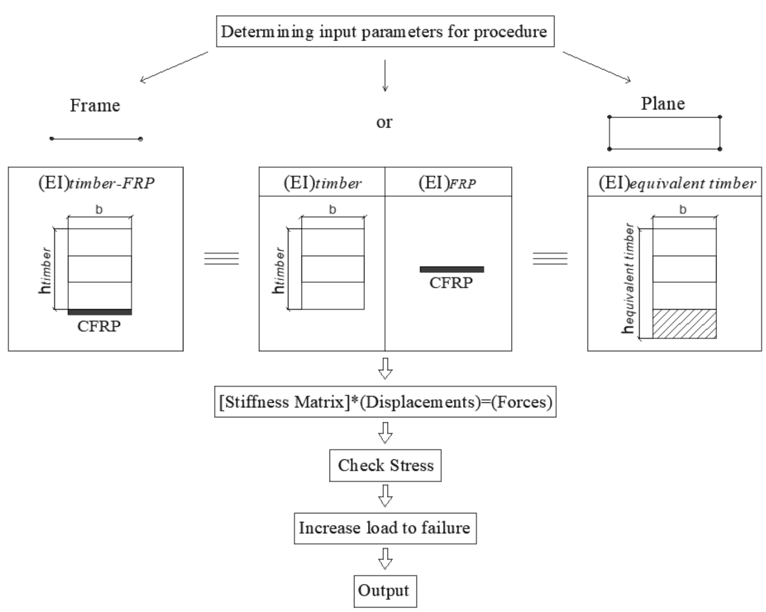

Figure 6. Transformation and analysis scheme.

4. Result and discussion

4.1. *Theoretical method results*

According to the theoretical model based on the study, the flexural moments and failure load values were calculated in the elastic and plastic states. In addition, the moment of inertia in the linear elastic state and the modulus of elasticity of the timber and the initial flexural stiffness (EI_t) of the beams were calculated. Theoretical method results of all the beams were given in Table 6.

Table 6. Results of the theoretical method.

Notation of beams	M_{ue} (N.mm)	M_{up} (N.mm)	P_e (kN)	P_p (kN)	EI_t (N.mm²) x10^{11}
T70	4.02	3.97	17.85	17.65	0.47
C35-T70	5.85	5.44	26.02	24.19	0.53
C70-T70	6.69	6.06	29.72	26.93	0.58
T50	2.87	2.84	12.75	12.60	0.34
C20-T50	4.54	4.16	20.17	18.47	0.40
C35-T50	5.25	4.66	23.35	20.70	0.44

Compared to T70 beams, when 0.5 mm thick CFRP plate is reinforced to half of the beam's cross-section, the increase in moment capacities is approximately 37%, and when the entire cross-section is wrapped, an increase of 53% has been observed. In addition, the increases in flexural stiffness are approximately 12% and 23%, respectively. Compared to T50 beams, when the 1 mm thick CFRP plate is reinforced to 40% of the cross-section of the beam, the increase in moment capacities is approximately 47%, and when it is reinforced to 70% of the cross-section, an increase of 64% is observed. In addition, the increases in flexural stiffness are approximately 19% and 31%, respectively.

When the results of the load carrying capacity of the T70 beam and the load carrying capacity of the C20-T50 beam are examined, it is seen that the values are close to each other. It can be said that CFRP reinforcement allows the use of smaller cross-section beams without compromising the load-carrying capacity of larger cross-section beams or with very little loss.

4.2. *FEM results*

The failure load (P_{FEM}), midspan vertical displacement (w_{FEM}), initial flexural stiffness (EI_{FEM}) and energy absorption capacities (E_{FEM}) values of beams obtained as a result of FEM are given in Table 7.

Table 7. Results of the FEM.

Notation of beams	P_{FEM} (kN)	w_{FEM} (mm)	EI_{FEM} (N.mm²) x10^{11}	E_{FEM} (kN.mm)
T70	16.61	16.11	0.45	144.89
C35-T70	23.60	23.94	0.53	310.62
C70-T70	25.71	24.12	0.57	395.45
T50	12.14	16.66	0.32	55.67
C20-T50	19.07	29.35	0.41	158.58
C35-T50	21.07	30.38	0.45	208.33

The initial flexural stiffness and energy absorption capacities of the beams were calculated using the load-midspan vertical displacement graphs given in Figures 7 and 8. The graphs contain the load and displacement values of the beams up to the failure load values. It is seen that the load carrying capacity, initial flexural stiffness, and energy absorption capacities of reinforced beams increase compared to unreinforced beams.

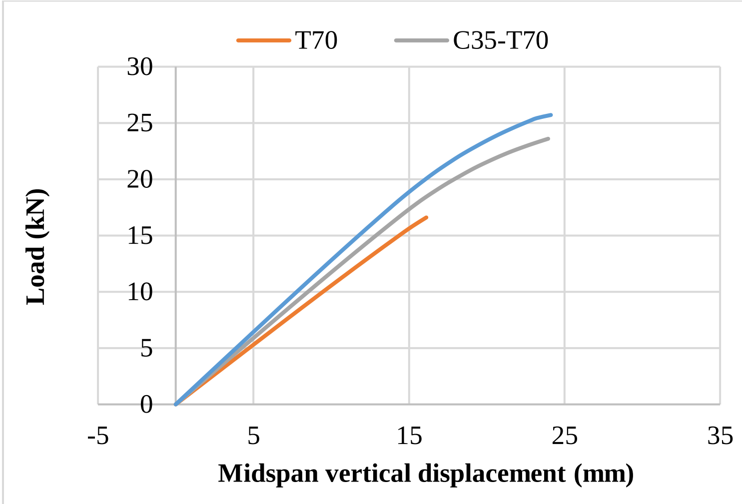

Figure 7. Load-midspan vertical displacement graphs of Group 1 beams.

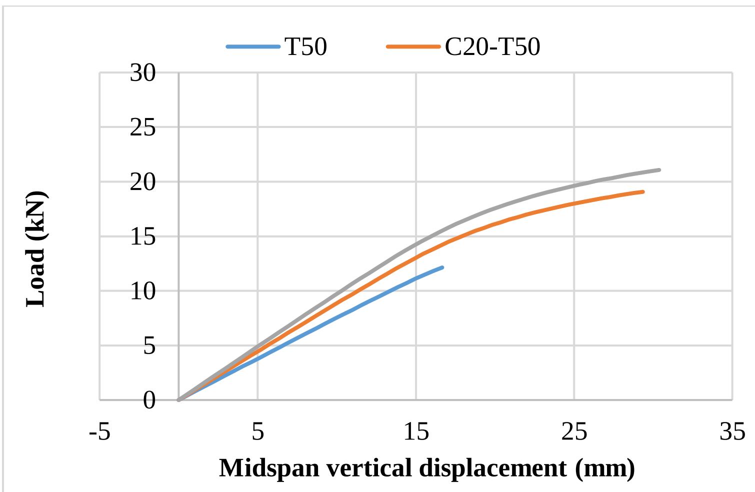

Figure 8. Load-midspan vertical displacement graphs of Group 2 beams.

Looking at the graphs of the unreinforced beams for two groups, the plastic state is limited to very small area until their load carrying capacity, and there is an almost linear behaviour. However, the load carrying continuity of the CFRP plate reinforced beams in the plastic region is clearly seen. In reinforced elements, after the timber lost its tensile capacity, the element reached collapse by using its compressive capacity as a result of the applied reinforcement. Also, it can be seen from the graphs that the rigidity of the system has increased.

The equivalent plastic strain (PEEQ) is used as a response to the permanent deformation of the part due to external loads. PEEQ changes of Group 1 and Group 2 beams are given in Figures 9 and 10, respectively.

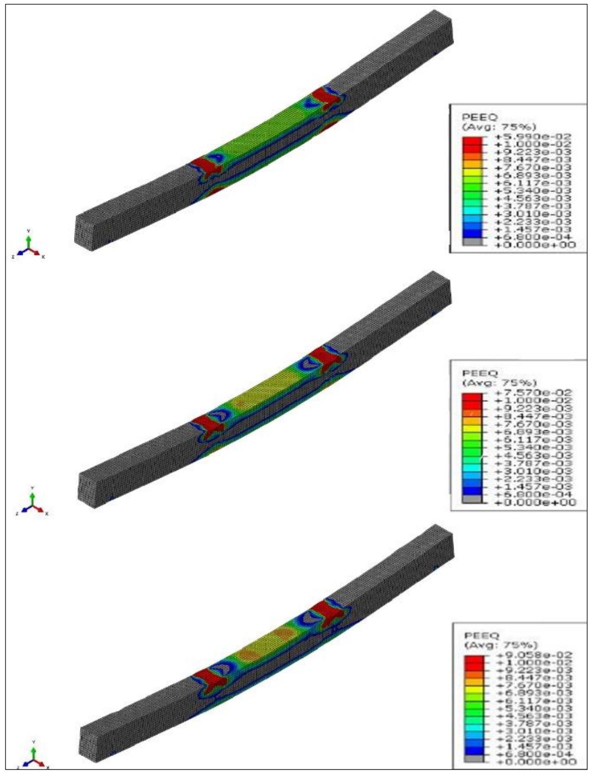

Figure 9. The equivalent plastic strain of Group 1 beams.

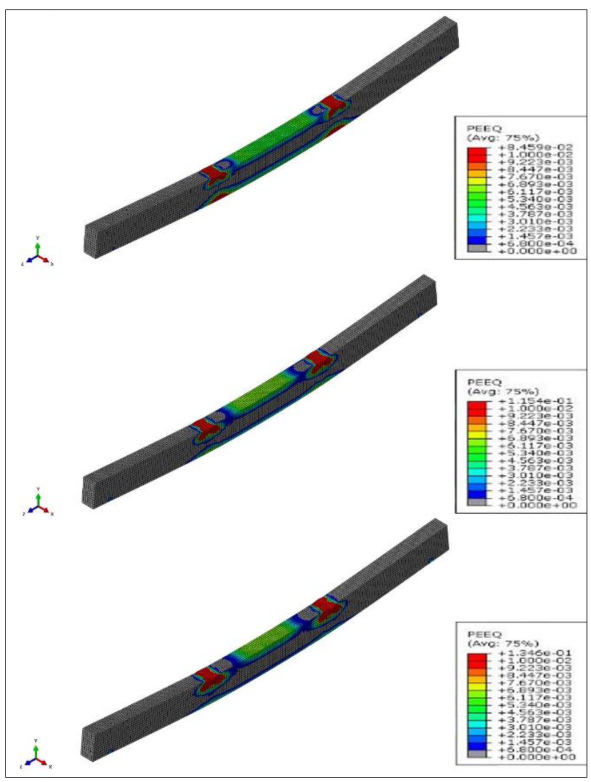

Figure 10. The equivalent plastic strain of Group 2 beams.

4.3. *Codes and standards results*

With these models, only elastic failure load can be obtained. Neither plastic stress nor plastic deformation can be obtained with these models. Thus, the determined failure mode happened in the elastic region. All beams failed under the load due to tensile stress. In addition, plane stress analysis showed compressive stress was the load application region. Results were given in Tables 8 and 9. Deformed shapes of models were given in Figure 11.

Table 8. Results of elastic frame analysis.

Notation of beams	Failure Load (kN)	Midspan Vertical Displacement (mm)	Initial Flexural Stiffness (N.mm²) x10^{11}
T70	11.0	12.32	0.47
C35-T70	18.7	9.72	1.00
C70-T70	19.4	10.47	1.82
T50	9.0	13.32	0.34
C20-T50	17.3	9.28	1.03
C35-T50	26.3	7.91	1.95

Table 9. Results of elastic plane analysis.

Notation of beams	Failure Load (kN)	Midspan Vertical Displacement (mm)	Initial Flexural Stiffness (N.mm²) x10^{11}
T70	16.5	5.00	0.47
C35-T70	28.5	5.35	1.00
C70-T70	40.0	4.00	1.82
T50	12.3	5.13	0.34
C20-T50	19.0	3.27	1.03
C35-T50	42.0	4.20	1.95

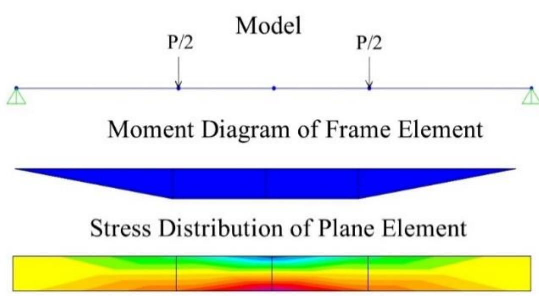

Figure 11. Linear model and results.

5. Conclusion

The failure load and initial flexural stiffness values of the beams analysed according to the theoretical, finite element method and code-based solution were given in Table 10 comparatively. As a result of the numerical approaches, the maximum loads and initial flexural stiffnesses that the CFRP plates reinforced in the tensile zone can carry compared to the reference beams increased in all three methods.

Although the theoretical and finite element method results converge, the finite element method results are of lower value. This is because the principles of interaction and fracture mechanics between laminate layers are taken into account in finite element analysis, but they are neglected in the theoretical method. In both models, the capacity increases caused by the CFRP reinforcement in the reference beams were similar. This was due to the similar approach of both models in modelling the interface between CFRP and timber. In addition, initial flexural stiffness was similar in both methods. Accordingly, it can be said that fracture mechanics and interlayer interaction are important in stress analysis rather than system displacements.

Code-based analysis yielded quite different results from theoretical and finite element analysis. The values found in the frame analyses were different from the theoretical and finite element results compared to the plane stress analyses. While plane-stress elements converged more in stress analysis, frame elements gave more realistic results in system displacements. Parallel to this, the frame element analyses converge more to the theoretical and finite element results at the initial flexural stiffness values.

As a result of the calculations, it is seen that nonlinear behaviour is important in the analysis of timber elements reinforced with fibre polymer materials. Fundamental criteria such as load carrying capacity, behaviour and system displacements can best be studied with the finite element method. On the other hand, while the theoretical method showed differences in load carrying capacity at low orders, it gave parallel results to the finite element method in displacement and behaviour. Moreover, considering that the frame model was more successful in displacements and plane-stress elements in stress analysis, it can be said that the modelling technique is the most important step in determining behaviour. Even in plain-stress analysis with linear analysis, the behaviour gave similar results with finite elements and plastic compressive stresses could be detected in the loading zones (Figures 9, 10 and 11). In order to determine the failure load correctly, nonlinearities such as plastic behaviour and interaction take up a place that cannot be neglected. Within the framework of these results, it can be said that the determination of material parameters and modelling method is not sufficient for reliable studies for real applications such as codes. Failure loads and displacements could not be determined with sufficient accuracy unless plastic behaviour was included in the analysis. Therefore, the determination of accurate analysis data by empirical, analytical or another similar method can provide a positive improvement in the examination of reinforcement applications of timber elements with fibre polymers. In Table 10, the results that the methods allow to examine are given comparatively.

Table 10. Comparison of theoretical, code-based and FEM analysis results.

Methods	Failure loads	Midspan vertical displacement	Application limits	Ease to use
Theoretical	Realistic	Adequate	Single element	Medium
Finite element	Realistic	Adequate	Global to local	Difficult
Elastic frame	Incorrect	Incorrect	Global	Simple
Plain stress	Incorrect	Incorrect	Global to local	Simple

As can be seen from Table 10, the finite element method is the most convenient method. On the other hand, while the theoretical method can give results with sufficient accuracy in single element investigations, it is not sufficient in real applications. Because, local behaviour cannot be detected here. Similarly, since the whole system cannot be handled together, it will be insufficient alone in daily applications. Even if the whole system, such as the structure, is considered together in the frame and plane stress analysis, the results do not give results with adequate accuracy. The finite element method is almost impossible in terms of practical applications. On the other hand, the development of a method in which the theoretical and plane stress method can be evaluated together can be suggested as the most convenient solution in terms of strengthening timber structures.

Author's contributions: All authors contributed to study conception. Main manuscript prepared by Dilan Cankal and Gokhan Sakar. All authors edited the manuscript. Huseyin Kursat Celik prepared the code-based part and the relevant part of the manuscript.

Funding: The authors received no financial support for this article.

Conflicts of interest: The authors declare that they have no conflict of interest.

References

Abrate, S. (2008). Criteria for yielding or failure of cellular materials. Journal of Sandwich Structures and Materials, 10(1), 5-5. https://doi.org/10.1177/1099636207070997

ACI Committee 440. (2017). Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. Farming Hills, Michigan, U.S.A.

Alhayek, H., & Svecova, D. (2012). Flexural stiffness and strength of GFRP-reinforced timber beams. Journal of Composites for Construction, 16(3), 245-252. https://doi.org/10.1061/(ASCE)CC.1943-5614.0000261

Ali, Y. A. Z. (2018). Flexural behavior of FRP strengthened concrete-wood composite beams. Ain Shams Engineering Journal, 9(4), 3419-3424. https://doi.org/10.1016/j.asej.2018.06.003

Aloisio, A., Boggian, F., Sævareid, H. Ø., Bjørkedal, J., & Tomasi, R. (2023). Analysis and enhancement of the new Eurocode 5 formulations for the lateral elastic deformation of LTF and CLT walls. Structures, 47, 1940-1956. https://doi.org/10.1016/j.istruc.2022.11.129

American Wood Council. (2015). *NDS National Design Specification for Wood Construction: With Commentary*. American Wood Council, USA.

ANSYS C. (2011). User manual Release 12.1. ANSYS Inc.

Aydın, I., Çolak, S., Çolakoğlu, G., & Salih, E. (2004). A comparative study on some physical and mechanical properties of Laminated Veneer Lumber (LVL) produced from Beech (Fagus orientalis Lipsky) and Eucalyptus (Eucalyptus camaldulensis Dehn.) veneers. Holz Als Roh-Und Werkstoff, 62(3), 218-220. http://doi.org/10.1007/s00107-004-0464-3

Bakalarz M. M., Kossakowski, P.G. (2022). Ductility and stiffness of laminated veneer lumber beams strengthened with fibrous composites. Fibres 10(2). https://doi.org/10.3390/fib10020021

Bodig, J., & Jayne, B. A. (1982). Mechanics of wood and wood composites. Van Nostrand Reinhold, New York.

Borri, A., Corradi, M., & Grazini, A. (2005). A method for flexural reinforcement of old wood beams with CFRP materials. Composites Part B: Engineering, 36(2), 143-153. https://doi.org/10.1016/j.compositesb.2004.04.013

BSI. (2009). BS EN 338: 2009. Structural timber-strength classes.

Bulleit, W. M., Sandberg, L. B., & Woods, G. J. (1989). Steel-reinforced glued laminated timber. Journal of Structural Engineering, 115(2), 433-444. https://doi.org/10.1061/(ASCE)0733-9445(1989)115:2(433)

Computers and Structures Inc. (2015). SAP2000 v15. Integrated Finite Element Analysis and Design of Structures.

Corradi, M., Mouli Vemury, C., Edmondson, V., Poologanathan, K., & Nagaratnam, B. (2021). Local FRP reinforcement of existing timber beams. Composite Structures, 258, 113363. https://doi.org/10.1016/j.compstruct.2020.113363

Dagher, H. J., Kimball, T. E., Shaler, S. M., & Abdel-Magid, B. (1996). Effect of FRP reinforcement on low-grade eastern hemlock glulams. In Proceedings of the National Conference on Wood in Transportation Structures, 207-214.

Danielsson, H., & Gustafsson, P. J. (2014). Fracture analysis of glued laminated timber beams with a hole using a 3D cohesive zone model. Engineering Fracture Mechanics, 124-125, 182-195. https://doi.org/10.1016/j.engfracmech.2014.04.020

Dietsch, P., & Winter, S. (2012). Eurocode 5—future developments towards a more comprehensive code on timber structures. Structural Engineering International, 22(2), 223-231. https://doi.org/10.2749/101686612X13291382991001

Dietsch, P. (2016). Reinforcement of timber structures–a new section for Eurocode 5. In Proceedings of the World Conference on Timber Engineering WCTE 2016, Vienna, Austria.

Donadon, B. F., Mascia, N. T., Vilela, R., & Trautwein, L. M. (2020). Experimental investigation of glued-laminated timber beams with Vectran-FRP reinforcement. Engineering Structures, 202, 109818. https://doi.org/10.1016/j.engstruct.2019.109818

European Committee for Standardization. (2004). Eurocode 5: Design of Timber Structures - Part 1-1: General - Common rules and rules for buildings, Brussels.

Fiorelli, J., & Dias, A. A. (2003). Analysis of the strength and stiffness of timber beams reinforced with carbon fibre and glass fibre. Materials Research, 6(2), 193-202. https://doi.org/10.1590/S1516-14392003000200014

Fiorelli, J., & Dias, A. A. (2011). Glulam beams reinforced with FRP externally-bonded: Theoretical and experimental evaluation. Materials and Structures, 44(8), 1431-1440. https://doi.org/10.1617/s11527-011-9708-y

Fortino, S., Zagari, G., Mendicino, A. L., & Dill-Langer, G. (2012). A simple approach for FEM simulation of Mode I cohesive crack growth in glued laminated timber under short-term loading. Journal of Structural Mechanics, 45(1), 1-20.

Gáborík, J., Gaff, M., Ruman, D., Záborsky, V., Kašíčková, V., & Sikora, A. (2016). Adhesive as a factor affecting the properties of laminated wood. BioResources, 11(4), 10565-10574.

Gentile, C., Svecova, D., & Rizkalla, S. H. (2002). Timber beams strengthened with GFRP bars: development and applications. Journal of Composites for Construction, 6(1), 11-20. https://doi.org/10.1061/(ASCE)1090-0268(2002)6:1(11)

Gilfillan, J., Gilbert, S., & Patrick, G. (2001). The improved performance of home-grown timber glulam beams using fibre reinforcement. Journal of the Institute of Wood Science, 15(6), 307-317.

Glišović, I., Pavlović, M., Stevanović, B., & Todorović, M. (2017). Numerical analysis of glulam beams reinforced with CFRP plates. Journal of Civil Engineering and Management, 23(7), 868-879. https://doi.org/10.3846/13923730.2017.1341953

Glišović, I., Stevanović, B., Todorović, M., & Stevanović, T. (2016). Glulam beams externally reinforced with CFRP plates. Wood Research, 61(1), 141-154.

Gustafsson, P. J. (2003). Fracture perpendicular to grain–structural applications. In T. Sh. J. Larsen (Ed.): Timber Engineering, John Wiley & Sons Ltd., pp. 103-130.

Haiman, M., & Žagar, Z. (2002). Strengthening timber glulam beams with FRP plates. In Wood in Construction Industry: Prospectives of Reconstruction. International Conference Proceedings, Zagreb, Croatia, 25-34.

Hill, R. (1948). A theory of the yielding and plastic flow of anisotropic metals. Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 193(1033), 281-297. https://doi.org/10.1098/rspa.1948.0045

Hoseinpour, H., Valluzzi, M. R., Garbin, E., & Panizza, M. (2018). Analytical investigation of timber beams strengthened with composite materials. Construction and Building Materials, 191, 1242-1251. https://doi.org/10.1016/j.conbuildmat.2018.10.014

İşleyen, Ü. K., Ghoroubi, R., Mercimek, Ö., Anil, Ö., & Erdem, R. T. (2021). Behavior of glulam timber beam strengthened with carbon fibre reinforced polymer strip for flexural loading. Journal of Reinforced Plastics and Composites, 40(17-18), 665-685. https://doi.org/10.1177/0731684421997924

Issa, C. A., & Kmeid, Z. (2005). Advanced wood engineering: glulam beams. Construction and Building Materials, 19(2), 99-106. https://doi.org/10.1016/j.conbuildmat.2004.05.013

Johns, K. C., & Lacroix, S. (2000). Composite reinforcement of timber in bending. Canadian Journal of Civil Engineering, 27(5), 899-906. https://doi.org/10.1139/l00-017

Khelifa, M., Auchet, S., Méausoone, P. J., & Celzard, A. (2015). Finite element analysis of flexural strengthening of timber beams with Carbon Fibre-Reinforced Polymers. Engineering Structures, 101, 364-375. https://doi.org/10.1016/j.engstruct.2015.07.046

Kim, Y. J., & Harries, K. A. (2010). Modeling of timber beams strengthened with various CFRP composites. Engineering Structures, 32(10), 3225-3234. https://doi.org/10.1016/j.engstruct.2010.06.011

Lacroix, D. N., & Doudak, G. (2018). Experimental and analytical investigation of FRP retrofitted glued-laminated beams subjected to simulated blast loading. Journal of Structural Engineering, 144(7), 04018089. https://doi.org/10.1061/(ASCE)ST.1943-541X.0002084

Lee, I. H., Song, Y. J., Song, D. B., & Hong, S. I. (2019). Results of delamination tests of FRP- and steel-plate-reinforced larix composite timber. Journal of the Korean Wood Science and Technology, 47(5), 655-662. https://doi.org/10.5658/WOOD.2019.47.5.655

Lee, M. J., Cho, T. M., Kim, W. S., Lee, B. C., & Lee, J. J. (2010). Determination of cohesive parameters for a mixed-mode cohesive zone model. International Journal of Adhesion and Adhesives, 30(5), 322-328. https://doi.org/10.1016/j.ijadhadh.2009.10.005

Li, Y. F., Xie, Y. M., & Tsai, M. J. (2009). Enhancement of the flexural performance of retrofitted wood beams using CFRP composite sheets. Construction and Building Materials, 23(1), 411-422. https://doi.org/10.1016/j.conbuildmat.2007.11.005

Lu, W., Ling, Z., Geng, Q., Liu, W., Yang, H., & Yue, K. (2015). Study on flexural behaviour of glulam beams reinforced by Near Surface Mounted (NSM) CFRP laminates. Construction and Building Materials, 91, 23-31. https://doi.org/10.1016/j.conbuildmat.2015.04.050

McConnell, E., McPolin, D., & Taylor, S. (2014). Post-tensioning of glulam timber with steel tendons. Construction and Building Materials, 73, 426-433. https://doi.org/10.1016/j.conbuildmat.2014.09.079

Meier, U. (1995). Strengthening of structures using carbon fibre/epoxy composites. Construction and Building Materials, 9(6), 341-351. https://doi.org/10.1016/0950-0618(95)00071-2

Micelli, F., Scialpi, V., & la Tegola, A. (2005). Flexural reinforcement of glulam timber beams and joints with carbon fibre-reinforced polymer rods. Journal of Composites for Construction, 9(4), 337-347. https://doi.org/10.1061/(ASCE)1090-0268(2005)9:4(337)

Morales-Conde, M. J., Rodríguez-Liñán, C., & Rubio-De Hita, P. (2015). Bending and shear reinforcements for timber beams using GFRP plates. Construction and Building Materials, 96, 461-472. https://doi.org/10.1016/j.conbuildmat.2015.07.079

National Research Council of Italy. (2007). Guidelines for the design and construction of externally bonded FRP systems for strengthening existing structures, Rome.

Raftery, G. M., Harte, A. M., & PR, And. (2009). Bonding of FRP materials to wood using thin epoxy glue lines. International Journal of Adhesion and Adhesives, 29(5), 580-588. http://dx.doi.org/10.1016/j.ijadhadh.2009.01.004

Raftery, G. M., & Harte, A. M. (2011). Low-grade glued laminated timber reinforced with FRP plate. Composites Part B: Engineering, 42(4), 724-735. https://doi.org/10.1016/j.compositesb.2011.01.029

Raftery, G. M., & Harte, A. M. (2013). Nonlinear numerical modelling of FRP reinforced glued laminated timber. Composites Part B: Engineering, 52, 40-50. http://dx.doi.org/10.1016/j.compositesb.2013.03.038

Systèmes, D. (2012). Abaqus/Standard Version 6.12-2. Computer program. Dassault Systèmes Simulia Corp. Providence.

Theiler, M., Frangi, A., & Steiger, R. (2013). Strain-based calculation model for centrically and eccentrically loaded timber columns. Engineering Structures, 56, 1103-1116. https://doi.org/10.1016/j.engstruct.2013.06.032

Thorhallsson, E. R., Hinriksson, G. I., & Snæbjörnsson, J. T. (2017). Strength and stiffness of glulam beams reinforced with glass and basalt fibres. Composites Part B: Engineering, 115, 300-307. https://doi.org/10.1016/j.compositesb.2016.09.074

Timbolmas, C., Bravo, R., Rescalvo, F. J., & Gallego, A. (2022). Development of an analytical model to predict the bending behavior of composite glulam beams in tension and compression. Journal of Building Engineering, 45, 103471. https://doi.org/10.1016/j.jobe.2021.103471

Tran, V. D., Oudjene, M., & Méausoone, P. J. (2014). FE analysis and geometrical optimization of timber beech finger-joint under bending test. International Journal of Adhesion and Adhesives, 52, 40-47. http://dx.doi.org/10.1016/j.ijadhadh.2014.03.007

Wacker, J. P., & Groenier, J. (2010). Comparative analysis of design codes for timber bridges in Canada, the United States, and Europe. Transportation research record, 2200(1), 163-168. http://dx.doi.org/10.3141/2200-19

Wang, C. M., Reddy, J. N., & Lee, K. H. (2000). Shear deformable beams and plates: relationships with classical solutions. Elsevier Science Ltd. https://doi.org/10.1016/B978-0-08-043784-2.X5000-X

Yahyaei-Moayyed, M., & Taheri, F. (2011). Experimental and computational investigations into creep response of AFRP reinforced timber beams. Composite Structures, 93(2), 616-628. https://doi.org/10.1016/j.compstruct.2010.08.017

Yang, H., Liu, W., Lu, W., Zhu, S., & Geng, Q. (2016). Flexural behavior of FRP and steel reinforced glulam beams: Experimental and theoretical evaluation. Construction and Building Materials, 106, 550-563. http://dx.doi.org/10.1016/j.conbuildmat.2015.12.135