Exponential Model for Damage Accumulation in Closed Cell Aluminum Foams

Modelo exponencial para la acumulación de daño en espumas de aluminio de células cerradas

Hernán Pinto (Main and Contact Author)
Pontificia Universidad Católica de Valparaíso, Facultad de Ingeniería,
Escuela de Ingeniería en Construcción, Chile
Avenida Brasil 2147, Valparaíso, Chile
+56-32-2273828
hernan.pinto@ucv.cl

Álvaro Peña
Pontificia Universidad Católica de Valparaíso, Facultad de Ingeniería,
Escuela de Ingeniería en Construcción, Chile.
alvaro.pena@ucv.cl

Ignacio Aravena
Pontificia Universidad Católica de Valparaíso, Chile
ignacio.aravena@gmail.com

Manuscript Code: 552
Date of Reception/Acceptance: 25.11.2014/01.06.2015

Abstract
In this paper the authors propose a new damage accumulation model based on an exponential approach for aluminum closed cell foam under fully reversed cyclic loading. The model developed need as an input, the information about the fatigue behavior of the material and the definition of the failure criterion. In this research, the failure criterion considered is the proposed by Ingraham et al. (2009), and the model to analyze the fatigue behavior is the well-known statistical Weibull model because this model allow us to us directly the total strain amplitude instead of its plastic and elastic components to analyze the fatigue behavior of the material. The model proposed it is a very simple mathematical expression that will allow us to model the damage accumulation level only as a function of the total strain amplitude. Finally, the proposed model has been validated through a comparison of the experimental damage accumulation curves, generated with the previous experimental data published by Ingraham et al. (2009), and the curve generated by the proposed model. The results provide a very good fit between the proposed model and the experimental curves.

Keywords: Metal foams; Closed Cell, Damage Accumulation; Fatigue Behavior, Weibull Model.

Resumen
En este artículo de investigación, los autores proponen un nuevo modelo de acumulación de daño, basado en una aproximación exponencial, para espumas metálicas de aluminio de células cerradas sometidas a cargas cíclicas completamente reversibles. El modelo desarrollado, necesita dentro de sus datos de entrada información acerca del comportamiento a fatiga del material y la definición del criterio de falla del mismo. En esta investigación, los autores utilizaron el criterio de falla propuesto por Ingraham et al. (2009), y el modelo estadístico de Weibull para analizar el comportamiento a fatiga del material, ya que éste último permite la utilización de la amplitud de deformación total en vez de utilizar las componentes plásticas y elásticas de la deformación. El modelo exponencial permite modelar la acumulación de daño del material como función solamente de la amplitud total de deformación. Finalmente el modelo propuesto se ha validado mediante una comparación de las curvas de daños experimentales y las curvas obtenidas mediante la utilización del modelo desarrollado, siendo los resultados de la curva de daño generada por el modelo muy satisfactorios ya que reproducen la curva de daño experimental de una muy buena manera.mejorar sus pequeñas deficiencias.

Palabras Claves: Espumas metálicas, Células cerradas, Acumulación de daño, Comportamiento a Fatiga, Modelo Weibull

Introduction

Metal foams have a very good strength and stiffness to weight ratios (light material) that make them very interesting and desirable for several applications in different industries, such as the automotive and aerospace. Since its appearance, several researchers have conducted different research projects to determine the mechanical and structural characteristics and properties of the material, as a consequence of these efforts, researchers have determined that metal foams are a material that has several interesting characteristics that make the material very attractive for different industries, among these characteristic we can highlight that they have a low density, low thermal conductivity, good fire resistance, high impact absorption, are non toxic, non inflammable, and provide a high noise reduction among others.

In the last few years, and considering the characteristic previously mentioned, metal foams has been introduced into the construction industry, in particular, designer and engineers has incorporated the material into different uses such as noise barrier or as a sound absorption structures in highways and railways, as a material for flooring, ceilings, walls covering, facades, roofs and emergency exit doors, among others.

Another application is related to light weighing structural elements, for example for mobile bridges, and also it can be used for reducing the energy consumption of elevators by a lightweight construction, finally and due to its lightweight, they are easy to handle and can be installed without mechanical lifting equipment, that make the material very attractive for perfect locations as ceilings, walls and roofs.

Since the use and applications of this material is increasing into different industries in the last few years, it is necessary to provide a fully understanding of the mechanical properties and behavior of the material under different types of loads and combinations. Since the appearance of the metal foams, several researchers (Ashby et al. 2000; Gibson et al. 1997; McCullogh et al. 1999; Sugimura et al. 1999) have conducted very interesting investigations in order to obtain the characterization of their mechanical properties and behavior, especially for monotonic or unreversed cyclic loads (tension-tension or compression-compression).

The authors a of these previous research state that for tension-tension fatigue loads the material with increasing fatigue cycles, will progressively elongates until the separation of the material (approximately at an accumulated strain of 1% (Ashby et al., 2000; Gibson et al., 1997; McCullogh et al., 1999, Sugimura et al 1999.), on the other hand under compression-compression fatigue loading, the material response is a progressive shortening to larger plastic strains (50%) with the fatigue life defined as the number of cycles at the point where the strain accumulation accelerates on a log-log plot of strain versus number of cycles (around 2% of accumulated strain (Ashby et al., 2000; Gibson et al., 1997; McCullogh et al., 1999; Sugimura et al., 1999).

For fully reversed loading, the literature is not as extensive as in the previous case; however, some interesting research of the fatigue response of closed cell foams under fully reversed cyclic loading has been conducted and published by Ingraham et al

(2009) and Pinto et al. (2011). In this paper, we propose a new damage accumulation model for closed cell foam subjected to fully reversed cyclic loading, this new model it is based on an exponential approach, and it will allow us to obtain directly the damage accumulation level in a closed cell aluminum foam as a function of the number of cycles applied, the total strain amplitude, and the initial value of the damage accumulation. In order to achieve this goal, a fatigue analysis model and a failure criterion are necessary; the failure criterion considered and used in this research, corresponds to the criterion presented by Ingraham et al. (2009) where they defined the failure in a closed cell foam when the ratio of the pre peak compressive slope and the pre peak tensile slope reach some defined value; and the model considered for the fatigue analysis is the well known statistical Weibull model proposed by Castillo et al. (2001, 2006, 2008) and Pinto (2009).

The paper is structured as follows: Section 2 describes the state of the art where both; the statistical Weibull model and the failure criterion proposed by Ingraham et al. (2009) are presented. Section 3 developed the exponential damage accumulation model. Next, section 4 provides a practical application of the proposed model using experimental data already published in the literature (Ingraham et al. (2009)). Fifth section presents a discussion of the results obtained using the proposed model, and finally in the last section the conclusions of the work are summarized.

State of the Art

In order to generate the damage accumulation model, a fatigue analysis model and a failure criterion are necessary. In this section a brief presentation of the Statistical Weibull Model for fatigue analysis and the failure criterion previously developed by Ingraham et al. (2009) are going to be introduced.

Statistical Weibull model for fatigue life analysis.

As it is mentioned in the previous section, the fatigue model to analyze the behavior of the material under cyclic loading is the well-known statistical Weibull model. The selection of this model is because it presents several advantages over the classical Coffin Manson approach (Castillo, 2006), among these advantages we can state that allow us to use directly the total strain amplitude instead of its elastic and plastic components, and provide a statistical representation of the fatigue behavior (Pinto, 2009).

The statistical Weibull model arises from a full statistical and physical analysis of the conditions that constrain the model; these conditions are the weakest link principle, stability, limit behavior, limited range and compatibility. These conditions ensure that the fatigue life of a system is governed by the shortest fatigue life of its constituent components (weakest link), that the distribution chosen for the fatigue life is valid over a broad range of specimen sizes with only the parameters being modified to account for size difference (stability), that the model displays a lower bound on the fatigue life and a fatigue threshold if appropriate (limit behavior and limited range), and that the cdf's of the fatigue life with respect to applied strain and the applied strain with respect to fatigue life are consistent with one another (compatibility).

These conditions are motivated by rigorous mathematics and physical reasoning (Castillo, 2001). The only model that satisfies all the conditions previously described is the Weibull model (Pinto, 2009). Once the model is selected, the compatibility condition is applied to ensure that the cdf of the fatigue life, conditional upon the strain amplitude, is compatible with the cdf of the strain amplitude, conditional upon the number of cycles. [1]

$$E(N^*;\varepsilon_a^*) = F(\varepsilon_a^*;N^*) \qquad [1]$$

$$N^* = exp\left[B\frac{\lambda+\delta(-log(1-p))^{(1/\beta)}}{(log\varepsilon_a^* - C)}\right] \qquad [2]$$

From this condition and considering the Weibull cdf's, a system of functional equations is generated. This system of equations has already been solved by Castillo et al. (2001), and after some mathematical work, the lifetime N* can be obtained through Equation 3, where εa is the strain amplitude, p the probability of failure, B the threshold value of lifetime, C the endurance limit, β the Weibull shape parameter of the cdf, λ the scale factor, δ the parameter that defines the position of the corresponding zero-percentile curve. [2]

The percentile curves that result from Equation 2 are shown in Figure 1, where, the zero-percentile curve represents the minimum number of cycles to produce failure for different values of εa*εa, figure 1 also shows the graphical interpretation of the model parameters B, the threshold value of lifetime, means that for lower values of the lifetime, there will be no failure, regardless the value of the strain amplitude, and C, that corresponds to the endurance limit provide the threshold value of strain amplitude, this means that for lower strains amplitudes applied to the specimen there will be no failure. Finally, the figure shows how the model directly provides, through the percentile curves, a statistical description of the strain-life curves for a given material.

Failure Criterion

Figure 1. Percentile curves representing the relationship between lifetime, N* and strain amplitude, Ea, in the E-N field for fatigue model. Prepared by the authors, 2013.

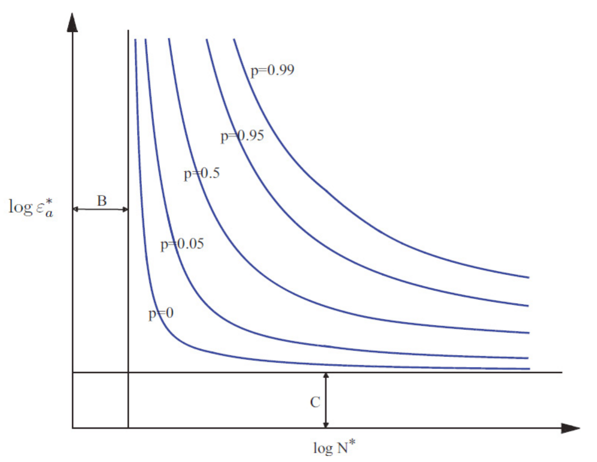
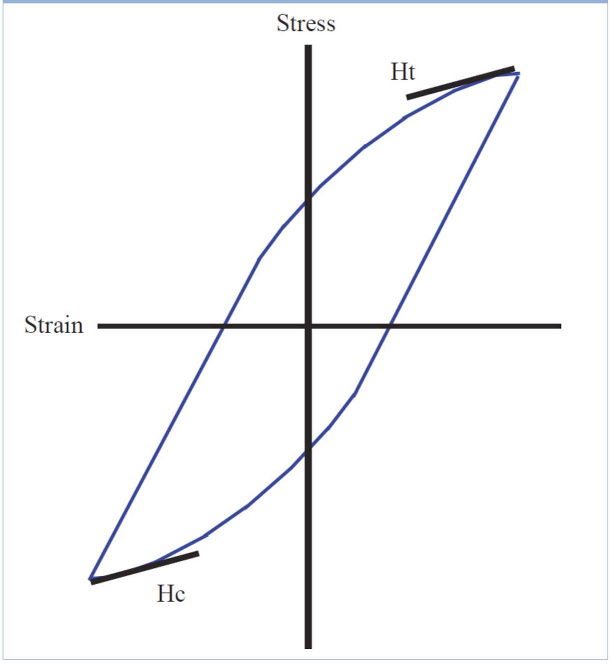


Figure 2 Schematic definitions of Hc and Ht according to Ingraham et al (2009). Prepared by the authors, 2013.



In order to be able to generate the damage accumulation model, in this research we use a set of published strain-life data and damage accumulation curves for closed cell aluminum foam already published by Ingraham et al. (2009). In metal foams, and in any kind of cellular solids type of materials, it is difficult to track standard measures of damage, such as the length of the dominant fatigue crack, In order to solve this issue; a new definition of a damage level is required. In this research, we consider the use of the innovative measure of damage state proposed by Ingraham et al. (2009) where they defined a damage measure value R as the ratio between the compressive pre-peak slope (Hc) and the tensile pre-peak slope (Ht) of the stress-strain curve (Figure 2).

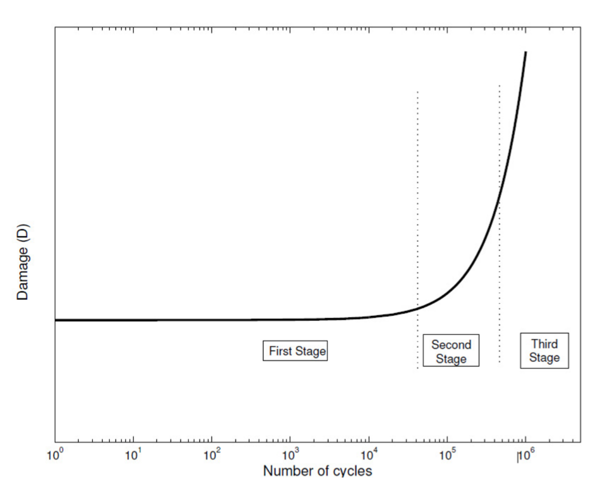
The authors propose that the ratio R = Hc / Ht, which is initially close to unity, increases as the material becomes more damaged, and failure occurs when R=1.5. This characterization provides a convenient scalar measure of the complicated damage process in metal foams, although it should be noted that without further testing its use should be restricted to the evaluation of damage under fully reversed cyclic loading.

Damage Accumulation Model

Damage accumulations models, provides an evolution of the damage curve of the material. The typical damage accumulation curve will have the behavior shown in figure 3, where 3 different regimes can be identified; the first regime consists a very long period stage with almost zero slope, which value will correspond to the initial value of damage of the material, the second stage the rate of damage accumulation starts to increase more rapidly, this increase of rate will lead the curve into the third stage where the maximum rate of damage accumulation is reached, as a consequence of this rate, the damage level will increase extremely fast until material failure.

A damage accumulation model can be describe by its most general expression as h=(x1, x2, x3, …, xn) where h is the function and xi are the variables of the problem. In this case, the variables involved in the determination of the damage are: the number of cycles (N), the initial damage (R0), and the total applied strain amplitude (ε_a), so the general form of the damage accumulation model has the form [3]

Figure 3 Schematic damage accumulation curve. Prepared by the authors, 2013.



$$R = h(N, \varepsilon_a, R_0) \qquad [3]$$

$$R = R_0 \exp(AN) \qquad [4]$$

$$A = \frac{log\left(\frac{1,5}{R_0}\right)}{exp\left[B + \frac{\lambda+\delta(-log(1-p))^{1/\beta}}{log\varepsilon_a^* - C}\right]} \qquad [5]$$

$$R = R_0 exp\left(\frac{log\left(\frac{1,5}{R_0}\right)}{exp\left[B + \frac{\lambda+\delta(-log(1-p))^{1/\beta}}{log\varepsilon_a^* - C}\right]} N\right) \qquad [6]$$

where R is the damage level, and h is the function that provides the level of damage. In this research, we consider that the h function will be exponential, so, introducing the exponential function into the general form of the damage accumulation, the model has the following general form. [4]

where A is the model parameter, R0 the initial damage state, and N the number of cycles. To obtain the parameter A of the damage accumulation model, the statistical Weibull model for the fatigue lifetime and the value considered by Ingraham et al.(2009) for the failure state (R*=1.5) are considered. After some mathematical work the following expression for the model parameters is obtained. [5]

Finally, after some mathematical work of the Equation 5, using the failure threshold value R*=1.5 and the strain-lifetime expressions for the statistical Weibull model (Equation 2), we obtain the general expression for the damage accumulation model. [6]

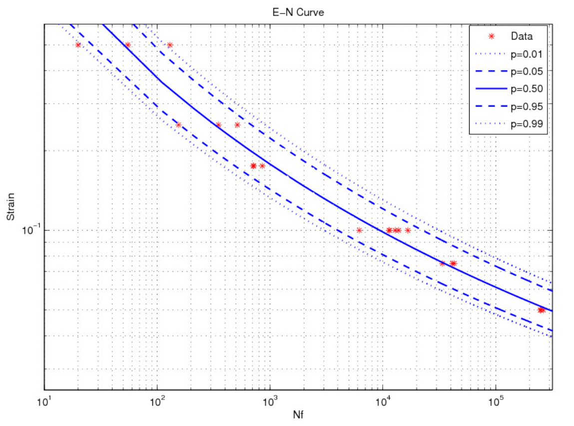
Application

This paper proposes a new damage accumulation model for closed cell aluminum foams. Since in this research, no experimental procedure was conducted, the material and data considered for the practical application of the model correspond to the material tested and results published by Ingraham et al. (2009).

Table 1. Strain-life data obtained experimentally. Source: Ingraham et al.(2009).

Total strain amplitude (%)	Cycles to Failure (N*)
0.050	251989
0.050	248336
0.050	264911
0.075	42924
0.075	33760
0.075	41407
0.100	13798
0.100	12871
0.100	11352
0.100	16700
0.100	11500
0.100	6200
0.175	7107
0.175	720
0.175	855
0.250	515
0.250	350
0.250	155
0.500	20
0.500	55
0.500	130

Figure 4 εa–N field curves of the experimental data (Ingraham et al. 2009) analyzed using the statistical Weibull model Prepared by the authors, 2013.



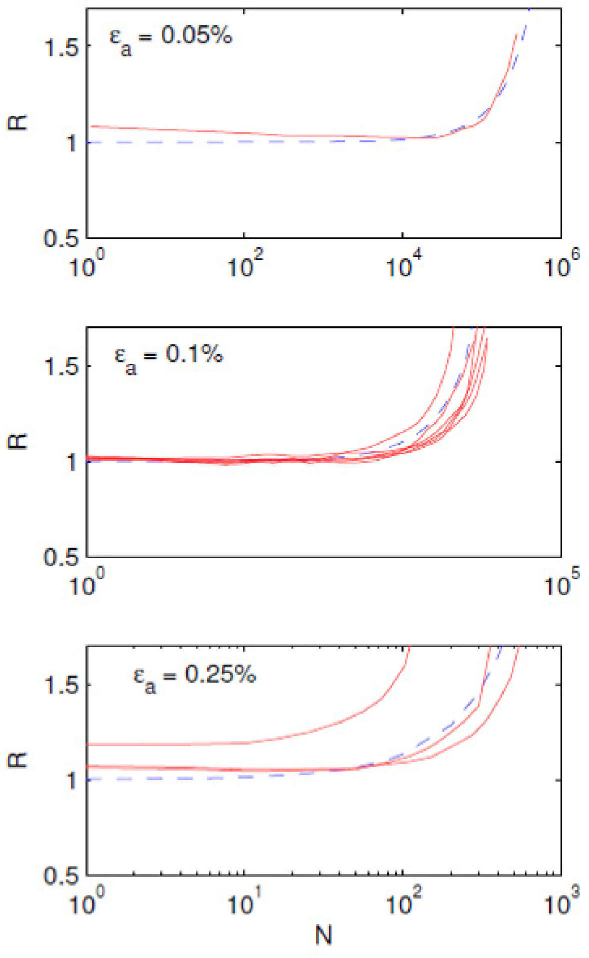
As a summary, we can state that the aluminum foam considered is commercially known as Alporas, with an average relative density of 8.7, cell sizes range from 1 to 8 mm. in diameter with an average cell measuring of 5 mm. approximately and with an average modulus of elasticity of 1.27 GPa in Tension and 1.25 GPa in compression (Ingraham et al. 2009) and the experimental results are summarized and presented in table 1. With the experimental data available (Table 1), the Statistical Weibull model was used to analyze the fatigue behavior of the material; as a result of the application of the model, the complete field of failure probability was obtained (Figure 4), and the Weibull parameters (B, C, β, λ, δ), were determined to be used as an input in the damage accumulation model, the value of the Weibull model parameters are presented in table 2.

Table 2. Weibull parameters of the experimental data (table 1) obtained from the Statistical Weibull model.

β	λ	δ	B	C
3.2997	8.9378	104.797	-12.9515	-7.4066

Once the Weibull parameters are determined (table 2), and the initial damage level is selected (R0=1.0), these values are incorporated into the general expression of the proposed model (Equation 7). At this point the proposed model has only the strain amplitude value as unkown, so for any value of total strain amplitude, we are able to generate the damage accumulation curve and determine the damage level for any number of cycle. In figure 4 the curves of the damage accumulation level of the aluminum foam using the proposed model (shown as dashed lines) are compared with the experimental results (shown as solid lines) obtained and previously published by Ingraham et al (2009) for six different total strain amplitudes levels (0.050%, 0.075%, 0.100%, 0.175%, 0.250% and 0.500%).

Figure 5a. Comparison of the performance of the Damage accumulation model (dashed lines) versus the experimental damage accumulation curves for different total strain amplitudes. Prepared by the authors, 2013.



Discussion

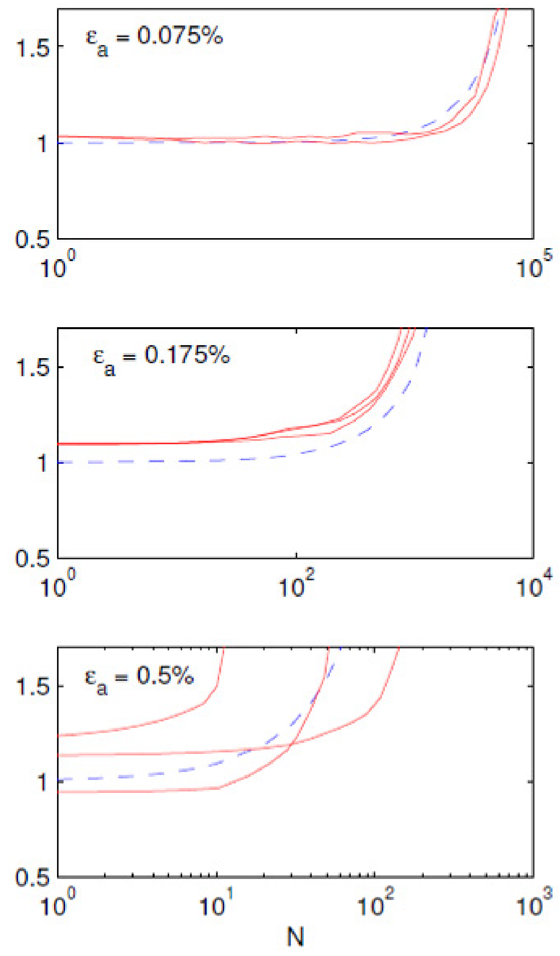
The model for damage accumulation level presented in this research, consider an exponential approach and is based on the statistical Weibull model for the lifetime analysis, and in the failure criterion proposed by Ingraham et al (2009), as a result we obtain a very simple mathematical expression that depends only on the total strain amplitude as the controlling parameter (Equation 7).

Analyzing the results presented in figure 4, the first thing that can be appreciated is that the curves generated by the model has the same general shape described in figure 3 as the typical behavior of the damage accumulation process. Also, we can appreciate that the results of the model (dashed lines) are very similar to the results obtained through the experimental campaign (solid line) developed by Ingraham et al (2009); in particular the slopes in the transitions zone, from stage 1 to stage 2 and from stage 2 to stage 3, and the behavior in the third stage are practically the same.

Some differences can be appreciated between the experimental curves and the model curves, however, these differences can be explained for the variability in the definition of the initial value of damage (R0), in the model we have consider the initial value of damage fixed and equal to 1, but the experimental results shown some scattered values of this parameter around 1 and not exactly one as it was considered in the proposed model. For the cases of strain amplitude 0.1% and 0.075% the experimental initial damage accumulation value was very close to one, and the better results of the exponential model where achieved, on the other hand, and as it was expected for higher strain amplitude (strain amplitudes equal to 0.25% and 0.5%) more differences where appreciated regarding the initial value of the damage accumulation. Nevertheless, the shape of the experimental and model curves are similar, the only difference is that the experimental curves are moved up or down in the damage level axis (y axis).

The use of the Weibull model to analyzed the fatigue behavior of the material provide several advantages, being the most important the possibility to use directly the total strain amplitude in order to run the exponential model instead of the use of the plastic components of the strain amplitude if a traditional Coffin Manson model will be considered. As a consequence of considering the Statistical Weibull model, the proposed exponential model will provide the damage accumulation level as a function of the total strain amplitude.
As it can be appreciated in figure 4, better results were obtained for the lower strain amplitudes, what make sense due to the less variability presented in the initial damage level, and also, because for lower strain amplitudes, the stresses are lower getting a longer fatigue life for the material, what will be traduced in a much more steady behavior of the damage level in comparison with higher strain amplitudes that will generate a shorter fatigue life of the material that are represented in a much more variable behavior of the damage level as it can be seen clearly through a simple comparison of the experimental curves for a total strain amplitude of 0.075% and 0.500% in figure 5, in the plots for lower strain amplitude can be appreciated that experimental curves are very similar between each other for the same strain amplitude (0.050%, 0.075%, 0.100, 0.175%), on the other hand when the total strain amplitude is increasing a higher variability of the experimental curve is appreciated (0.250% and 0.500%).

Figure 5b. Comparison of the performance of the Damage accumulation model (dashed lines) versus the experimental damage accumulation curves for different total strain amplitudes. Prepared by the authors, 2013.



Conclusions

A new damage accumulation model based on an exponential approach that includes the statistical Weibull model for the fatigue lifetime analysis and the failure criterion proposed by Ingraham et al (2009) is presented. The model, through a very simple mathematical procedure that fully integrate the failure criterion with the statistical model for the fatigue lifetime, allow us to determine the damage accumulation curves for closed cell aluminum foams subjected to a fully reversed cyclic load.

The advantage of using the statistical Weibull model to analyze the fatigue behavior, is that allows us to model statistically the fatigue behavior of metal foams under fully reversed cyclic loading conditions and it considers the use of the total strain amplitude instead of its plastic and elastic components needed to use the traditional fatigue analysis models.

The proposed exponential model for damage accumulation provides adequate predictions of the fatigue lifetime of closed cell aluminum foams subjected under fully reversed cyclic loading, and can be used directly with the total strain amplitude as the controlling parameter. The better results of the proposed model are obtained for the lower strain amplitudes due to two different reasons, the first one corresponds to the variability of the initial damage of accumulation level, considered equal to one in the proposed model, nevertheless, in the experimental results it can be appreciated that this initial damage level it is not necessary equal to one, especially for the higher strain amplitudes; the second reason is that for lower strain amplitudes, the behavior of the material for damage level is much more steady that in the case of higher strain amplitudes.

Regarding the set of experimental data available for this research, the proposed exponential model it is suitable to describe the damage accumulation level of the closed cell foams for total strain amplitudes lower than 0.250%. A further investigation must be conducted in order to incorporate the variability and randomness of the initial value of damage accumulation.

Acknowledgments

This research was financially supported by the Comisión Nacional de Investigatción Científica y Tecnológica (CONICYT) of Chile through the project FONDECYT de INICIACION Nº 11110365.

References

Ingraham, M. D., DeMaría C. J., Issen, K. A., & Morrison, D. J. (2009). Low Cycle Fatigue of aluminum foam. Material Science and Engineering A, 504, 150-156.

Ashby, M., Evans, A., Fleck, N., Gibson, L., Hutchinson, J., & Wadley, H. (2000). Metal foams a design guide. Boston, MA: Butterworth-Heinemann.

Gibson, L.G. (1997). Cellular Solids. (2nd Edition) Cambridge: Cambridge University Press

McCullogh, K.Y.G., Fleck, N.A. & Ashby, M.F. (1999). The stress-life fatigue behavior of aluminum alloy foams. Fatigue and Fracture Engineering Materials and structures, 23, 199-208.

Sugimura, Y., Rabiei, A., He, Y., Harte, A.M., Evans, A.G., & Fleck, N.A. (1999). Compression fatigue of a cellular Al alloy. Materials Science and Engineering A, 269, 38-48.

Pinto, H. & Arwade, S.R. (2011) Damage accumulation model for aluminum closed cell foams subjected to fully reversed cyclic loading. Fatigue and Fracture Engineering Materials and Structures, 34 (12), 1021-1234.

Castillo, E., Fernández-Canteli, A., Hadi, A.S. & López-Aenlle, M. (2006). A fatigue model with local sensitivity analysis. Fatigue and Fracture Engineering Materials and Structures. 30: 149-168.

Castillo, E., Fernández-Canteli, A., Pinto, H. & López-Aenlle, M. (2008). A general regression model for statistical analysis of strain life data. Materials Letters. 62, 3639-3642.